

Hong Kong Mathematics Olympiad (1999 – 2000)

Heat Event (Individual)

香港數學競賽 (1999 – 2000)

初賽項目(個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 設 $x = 0.\dot{1}\dot{7} + 0.0\dot{1}\dot{7} + 0.00\dot{1}\dot{7} + \dots$ ，求 x 的值。

Let $x = 0.\dot{1}\dot{7} + 0.0\dot{1}\dot{7} + 0.00\dot{1}\dot{7} + \dots$, find the value of x .

2. 解下列方程：

$$\frac{1}{x+12} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \dots + \frac{1}{(x+10)(x+11)} + \frac{1}{(x+11)(x+12)} = \frac{1}{4}.$$

Solve the following equation:

$$\frac{1}{x+12} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \dots + \frac{1}{(x+10)(x+11)} + \frac{1}{(x+11)(x+12)} = \frac{1}{4}.$$

3. 用數字 0、1、2、5 可以組成多少個能被 5 整除的三位數？(若數字不可以重複使用。)

Using digits 0, 1, 2, and 5, how many 3-digit numbers can be formed, which are divisible by 5? (If no digit may be repeated.)

4. 在圖一，有一個 4×3 的矩形蜘蛛網。若有一隻蜘蛛沿著網絲爬行。而其爬行方向祇可向東或向北。該蜘蛛由 A 點到 C 點共有多少種可能路徑？

Figure 1 represents a 4×3 rectangular spiderweb. If a spider walks along the web from A to C and it always walks either due East or due North. Find the total number of possible paths.

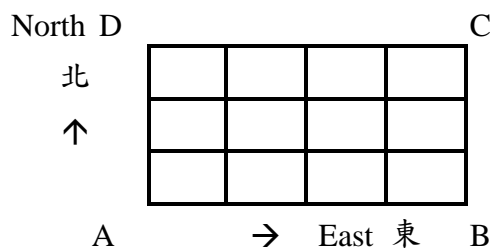


Figure 1
圖一

5. 在圖二，設 $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G = x^\circ$ ，求 x 的值。

In Figure 2, let $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G = x^\circ$, find the value of x .

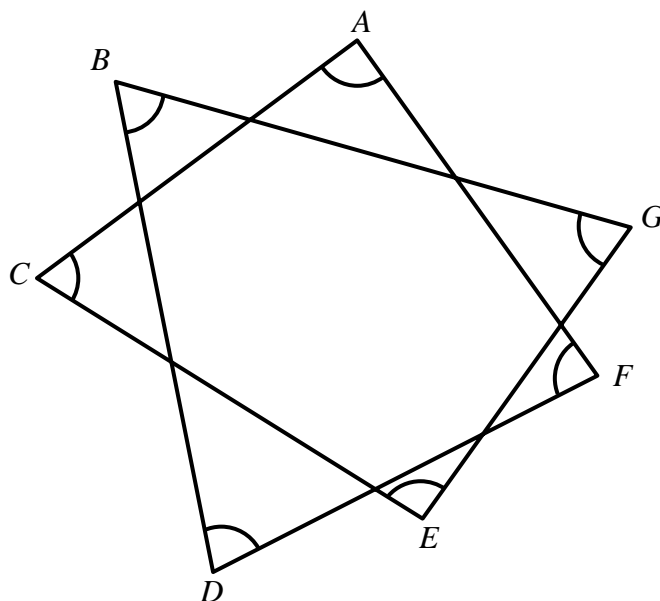


Figure 2

圖二

6. 於一白紙上，畫有 20 條直線。該 20 條直線，並沒有兩條或兩條以上是平行的，也沒有三條或三條以上的直線共點，問這 20 條直線最多可構成多少個交點？

Twenty straight lines were drawn on a white paper. Among them, no two or more straight lines are parallel; also no three or more than three straight lines are concurrent. What is the maximum number of intersections that these 20 lines can form?

7. 某一家庭有兩個孩子，已知其中一個孩子是女的，求該家庭的另一個孩子亦是女兒的概率是多少？(假設生男、生女的概率相等。)

In a family of 2 children, given that one of them is a girl, what is the probability of having another girl? (Assuming equal probabilities of boys and girls.)

8. 有一個六位數，其個位數字為「1」，若將該個位數字「1」移至十萬位，其原來的十萬位數字、萬位數字、千位數字、...皆向右順移一個位。新的六位數的值為原來的六位數的值的 $\frac{1}{3}$ ，求原來的六位數。

A particular 6-digit number has a unit-digit “1”. Suppose this unit-digit “1” is moved to the place of hundred thousands, while the original ten thousand-digit, thousand-digit, hundred-digit, ...are moved one digit place to the right. The value of the new 6-digit number is one-third of the value of the original 6-digit number. Find the original 6-digit number.

9. 求 $\frac{12\sin^2 48^\circ + 12\sin^2 42^\circ}{\sin 330^\circ \tan 135^\circ - \sin^2 48^\circ \sin^2 42^\circ \tan 180^\circ}$ 的值。

Find the value of $\frac{12\sin^2 48^\circ + 12\sin^2 42^\circ}{\sin 330^\circ \tan 135^\circ - \sin^2 48^\circ \sin^2 42^\circ \tan 180^\circ}$.

10. 求直線 $3x - y - 4 = 0$ 與點 $(2, 2)$ 的最短距離。

Find the shortest distance between the line $3x - y - 4 = 0$ and the point $(2, 2)$.